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National Aeronautics and Space Administration

Exergy Efficiency of Interplanetary Transfer Vehicles

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Thermodynamic Exergy

- **Thermodynamic exergy measures the useful work done by a system**
  - Provides the a full representation of all work (i.e., thermal, mechanical, kinetic, potential) done by a system
- **Rockets and spacecraft are thermodynamic systems**
- **Exergy Efficiency provides a full characterization of a vehicle's performance**
  - Provides a meaningful system characteristic to understand system interactions across multiple disciplines, subsystems and scales
  - Provides the ability to compare different systems performing similar missions

Exergy Balance Equations for a Rocket

Exergy Balance

$$\sum_{stages}\left[\Delta m_{propellant}\left(h_{prop}+\frac{V_e^2}{2}\right)\right]-X_{des}=$$

$$\sum_{stages}\left[\left(M_{vehicle,final}\frac{V^2_{vehicle,final}}{2}-M_{vehicle,initial}\frac{V^2_{vehicle,initial}}{2}\right)+\left(\frac{GM_EM_{vehicle,initial}}{r_{altitude,initial}}-\frac{GM_EM_{vehicle,final}}{r_{altitude,final}}\right)\right]$$

Simplifies to Orbital energy relationship during coast phases

$$\boldsymbol{X_{vehicle}}=E_{vehicle}=\left(M_{vehicle}\frac{V^2_{vehicle}}{2}-\frac{GxM_EM_{vehicle}}{r_{altitude}}\right)$$

Exergy Efficiency

$$\boldsymbol{\eta_{exergy}=1-\frac{X_{des}}{X_{expended}}=1-}\frac{X_{des}}{\boldsymbol{\Delta mprop\left(h_{prop}+\frac{V_e^2}{2}\right)}}$$

Mars Transfer Exergy Analysis

Reference State

- Reference state is determined by:
  - Vacuum of space (pressure, accounted for in engine Isp)
  - Thermal (Solar irradiance and planetary reflections) (assumed controlled by tank insulation and not considered in this model)
  - Solar and planetary gravitational effects
    - Planetary motions and masses contribute significant exergy to the rocket not provided by the rocket propulsion
      - Vehicle potential energy reference changes with respect to the planets and sun

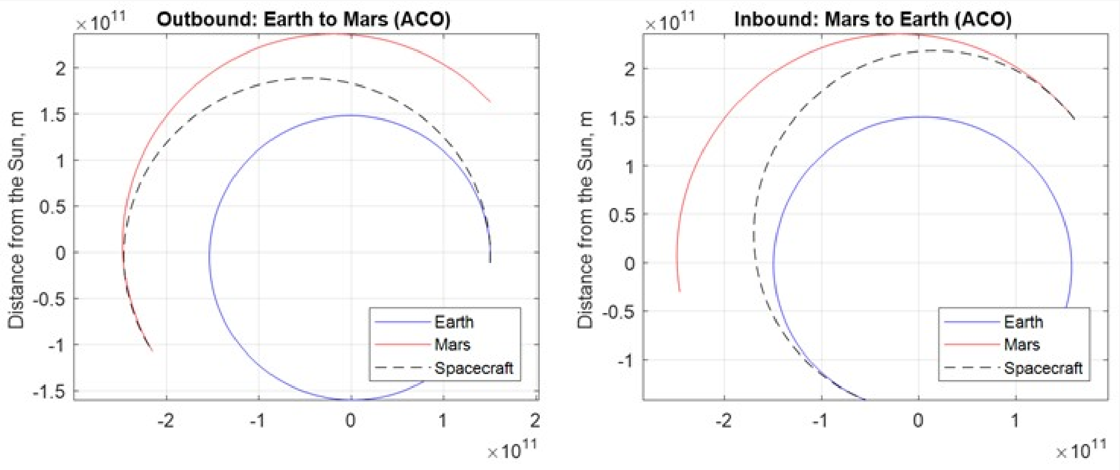
Solar dominates outside the sphere of influence (SOI) for any planet

Planet dominates inside its SOI, where the SOI radius is given as:

$$r_{SOI} = r_{sun,planet}\left(\frac{m_{planet}}{m_{sun}}\right)^{2/5}$$

Mission Design

- **Hohmann transfers for Earth to Mars**
- **11-month stay on the planet**
- **Hohmann transfer for Mars to Earth**
- **Total mission length on the order of two to three years.**



- **This trajectory contains four main burns:**
  - Trans-Mars injection (TMI),
  - Mars orbit insertion (MOI),
  - Trans-Earth injection (TEI), and
  - Earth orbit insertion (EOI).

- **Four different propulsion systems were analyzed**
  - Low enriched uranium (LEU) liquid hydrogen (LH2) nuclear thermal propulsion (NTP),
  - High enriched uranium (HEU) LH2 NTP,
  - LEU CH4 (methane) NTP, and
  - Chemical liquid oxygen (LO2)/LH2 system.

Engine and Thruster Characteristics

Main Engine Characteristics (Thrust/Isp Given)

- $\dot{m}_{propellant} = T_{engine}/(I_{sp} g_0)$

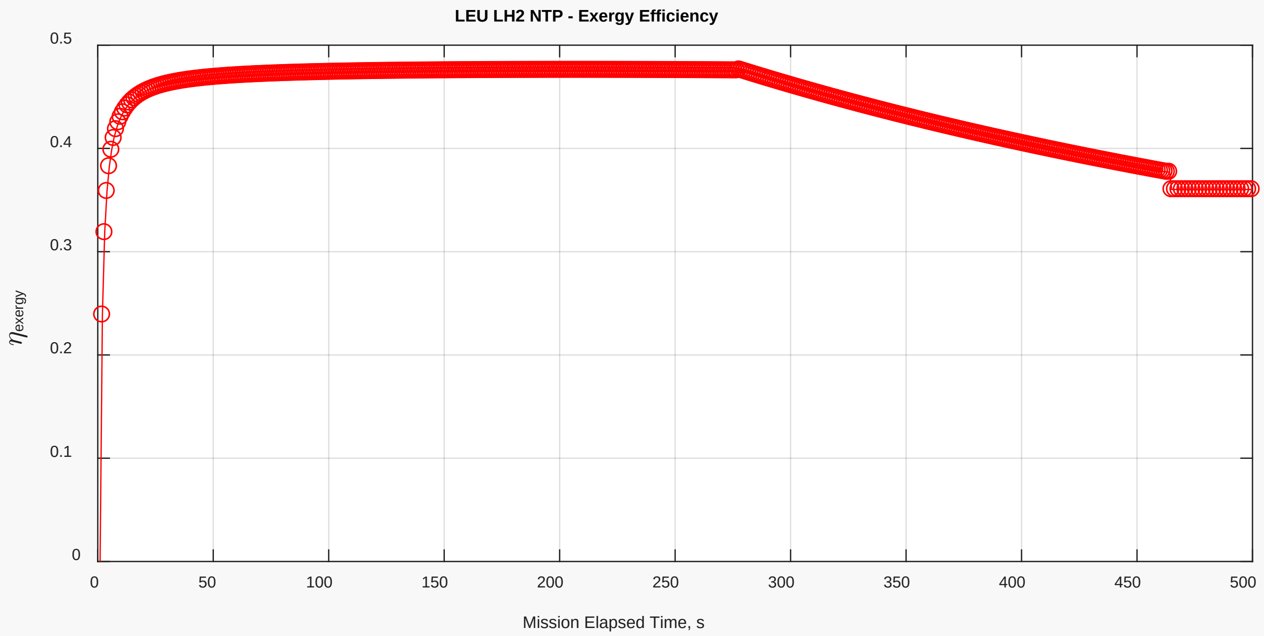
$$\Delta m_{prop} = \frac{m_0}{e^{\left(\frac{\Delta V \dot{m}}{F}\right)}} - m_0$$

RCS Characteristics

- Typical RCS values
  - $\dot{m}$ = 7 kg/s
  - I_{sp} = 291 s
- RCS burns are 40 m/s, fully tangential

TMI, First 500 s

- **RCS burns noticeably effect Exergy Efficiency**
  - Much lower Isp than main engines



Vehicle Position and Velocity

- ***Equations of position and motion relative to planetary horizons approaching SOI from interplanetary space***
  - $\vec{r}_{vehicle,planet} = \vec{r}_{vehicle,sun} - \vec{r}_{planet,sun}$
  - $\vec{V}_{vehicle,planet} = \vec{V}_{vehicle,sun} - \vec{V}_{planet,sun}$
  - $\varphi_{planetary\ horison} = \frac{\pi}{2} - acos\left(\frac{\vec{V}_{vehicle,planet}\vec{r}_{vehicle,planet}}{\left\|\vec{V}_{vehicle,planet}\right\|\left\|\vec{r}_{vehicle,planet}\right\|}\right)$
- **Time of SOI crossing**
  - $t_{SOI} = t_i + (t_f - t_i)\frac{\vec{r}_{SOI} - \left\|\vec{r}_{vehicle,planet,i}\right\|}{\left\|\vec{r}_{vehicle,planet,f}\right\| - \left\|\vec{r}_{vehicle,planet,i}\right\|}$
- ***Equations of position and motion relative to planets SOI***
  - $\vec{V}_{vehicle,planet,SOI} = \vec{V}_{vehicle,planet,i} + (t_{SOI} - t_i)\left(\frac{\vec{V}_{vehicle,planet,f} - \vec{V}_{vehicle,planet,i}}{t_f - t_i}\right)$
  - $\varphi_{horizon,SOI} = \varphi_{horizon,i} + (t_{SOI} - t_i)\left(\frac{\varphi_{horizon,f} - \varphi_{horizon,i}}{t_f - t_i}\right)$

SOI Reference Frame Transform

Interplanetary to Planetary Reference Frame Transform

- $\hat{\imath} = \dfrac{\vec{r}_{vehicle,planet,SOI}}{\left\|\vec{r}_{vehicle,planet,SOI}\right\|}$
- $\hat{k} = \dfrac{\hat{\imath} x \vec{V}_{vehicle,planet,SOI}}{\left\|\hat{\imath} x \vec{V}_{vehicle,planet,SOI}\right\|}$
- $\hat{\jmath} = \dfrac{\hat{k} x \hat{\imath}}{\|\hat{k} x \hat{\imath}\|}$
- $T_{Transform} = \begin{bmatrix} \hat{\imath}_X & \hat{\imath}_Y & \hat{\imath}_Z \\ \hat{\jmath}_X & \hat{\jmath}_Y & \hat{\jmath}_Z \\ \hat{k}_X & \hat{k}_Y & \hat{k}_Z \end{bmatrix}$

Planetary Orbit Entry Conditions

- $V_{SOI} = \left\|\vec{V}_{ship,planet,SOI}\right\|$
- $a_{transfer} = 1 / \left(\left(\dfrac{2}{r_{SOI}} \right) - \left(\dfrac{V_{SOI}^2}{\mu_{planet}} \right) \right)$

Planetary Parking Orbit

Parking Orbit Parameters

- Periapsis = 400 km above planetary surface
  - Well above atmospheric drag at Earth (similar to ISS) or Mars
    - Note aero-braking would require a lower periapsis on arrival (not addressed in these calculations)

- $e_{transfer} = 1 - \frac{r_{periapsis}}{a_{transfer}}$
- $V_{periapsis,transfer} = \sqrt{\mu_{planet}\left(\frac{2}{r_{periapsis}} - \frac{1}{a_{transfer}}\right)}$
- $V_{periapsis,parking} = V_{periapsis,transfer} - \Delta V$
- $a_{parking} = 1/\left(\left(\frac{2}{r_{periapsis}}\right) - \left(\frac{V^2_{periapsis,parking}}{\mu_{planet}}\right)\right)$
- $e_{parking} = 1 - \frac{r_{periapsis,parking}}{a}$
- $r_{apoapsis} = r_{periapsis}\left(\frac{1+e_{parking}}{1-e_{parking}}\right)$
- $\theta_{SOI} = \mathrm{acos}\left(\frac{a_{transfer}\left(1-e^2_{transfer}\right)-r_{SOI}}{r_{SOI}e_{transfer}}\right)$

- Apoapsis should be checked to ensure it stays within SOI.
  - Can iteratively solve the above equations stepping the apoapsis closer to the planet to achieve needed apoapsis.
- ΔV point thrust performed at SOI boundary to rotate flight path from hyperbolic transfer orbit to planetary parking orbit (i.e., patched conic solution)

Planetary Parking Orbit Burns

- **Parking Orbit burns calculated looking at vehicle position and velocity to maintain desired orbital trajectory with SOI intersection point**
  - $r_f = r_i + V_i \Delta t + \frac{1}{2} \dot{V}_i \Delta t^2$
  - $V_f = V_i + \dot{V}_i \Delta t$

- **Both departure parking orbits to SOI and arrival parking orbits can be calculated following this approach (forward to SOI or backward from SOI)**

- **Helio-Centric Parameters can be calculated from the planetary equations as**
  - $\vec{r}_{vehicle,sun} = \vec{r}_{planet,sun} + \left(T_{Transform} \vec{r}_{vehicle,planet}\right)$
  - $\vec{V}_{vehicle,sun} = \vec{V}_{planet,sun} + \left(T_{Transform} \vec{V}_{vehicle,planet}\right)$

Exergy Efficiency Equations

Vehicle Exergy

- KE: $m_f V_f^2 - m_i V_i^2 = \begin{cases} > 0 \\ < 0 \end{cases}$
- PE: $\frac{m_i}{r_i} - \frac{m_f}{r_f} = \begin{cases} > 0 \\ < 0 \end{cases}$

Mass	Velocity	ΔKE$_{step}$	Distance	ΔPE$_{step}$
$M_f = M_i$	$V_f > V_i$	$+$	$r_f > r_i$	$+$
$M_f = M_i$	$V_f < V_i$	$-$	$r_f < r_i$	$-$
$\begin{cases} M_f > M_i \\ M_f = XM_i \end{cases}$	$\begin{cases} V_f > V_i \\ V_f = ZV_i \end{cases}$	$+$	$\begin{cases} r_f > r_i \\ r_f = Yr_i \end{cases}$	$\begin{cases} + \ (Y > X) \\ - \ (Y < X) \end{cases}$
$\begin{cases} M_f > M_i \\ M_f = XM_i \end{cases}$	$\begin{cases} V_f < V_i \\ V_i = ZV_f \end{cases}$	$\begin{cases} - \ (Z^2 > X) \\ + \ (Z^2 < X) \end{cases}$	$\begin{cases} r_f < r_i \\ r_i = Yr_f \end{cases}$	$-$
$\begin{cases} M_f < M_i \\ M_i = XM_f \end{cases}$	$\begin{cases} V_f > V_i \\ V_f = ZV_i \end{cases}$	$\begin{cases} + \ (Z^2 > X) \\ - \ (Z^2 < X) \end{cases}$	$\begin{cases} r_f > r_i \\ r_f = Yr_i \end{cases}$	$+$
$\begin{cases} M_f < M_i \\ M_i = XM_f \end{cases}$	$\begin{cases} V_f < V_i \\ V_i = ZV_f \end{cases}$	$-$	$\begin{cases} r_f < r_i \\ r_i = Yr_f \end{cases}$	$\begin{cases} - \ (Y > X) \\ + \ (Y < X) \end{cases}$

- Where
  - X, Y, Z ≥ 1

Exergy Efficiency Equations

Vehicle Change in Exergy

- $\Delta KE_{step} = \frac{S}{2}\left|m_f V_f^2 - m_i V_i^2\right|$
- $\Delta PE_{step} = S\mu \left|\frac{m_i}{r_i} - \frac{m_f}{r_f}\right|$
  - Where S is sign given from table on previous chart

Thrust Exergy

- $X_{exp} = \Delta m_{propellant}\left(h_{prop} + \frac{V_e^2}{2}\right)$

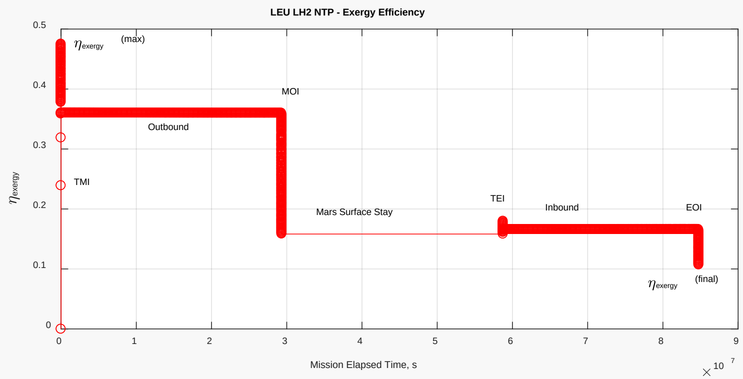
Vehicle Exergy Balance

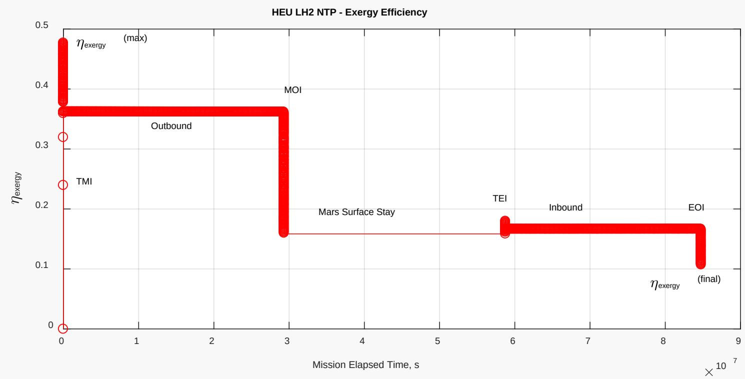
- $X_{des} = X_{exp} - \sum \Delta KE_{step} - \sum \Delta PE_{step}$

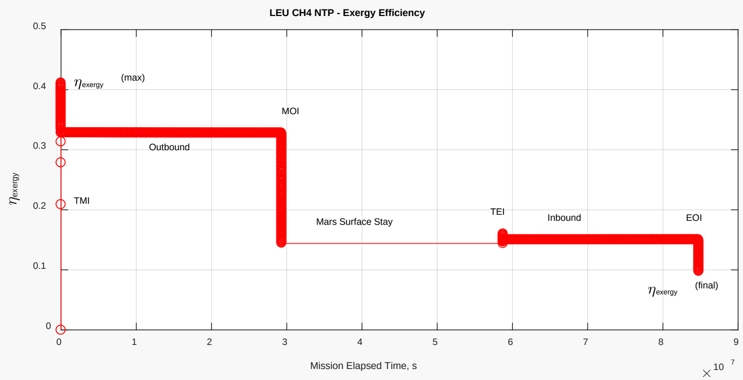
Vehicle Exergy Efficiency

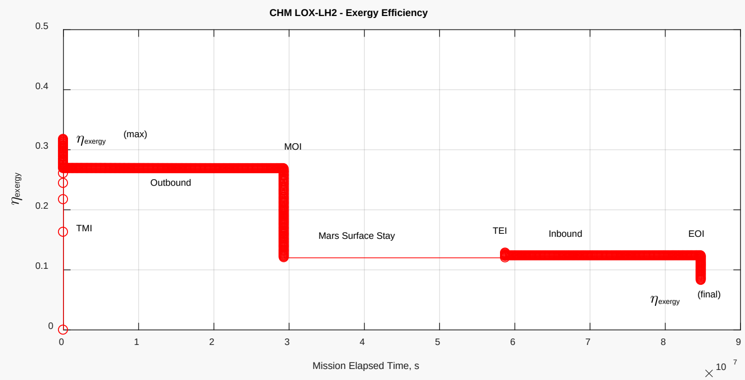
- $\eta_{exergy} = \frac{\Delta m_{propellant}\left(h_{prop}+\frac{V_e^2}{2}\right) - X_{des}}{\Delta m_{propellant}\left(h_{prop}+\frac{V_e^2}{2}\right)} = 1 - \frac{X_{des}}{\Delta m_{propellant}\left(h_{prop}+\frac{V_e^2}{2}\right)}$

Exergy Efficiency









Results

- η_{exg} roughly scales directly with I_{sp} and inversely with total vehicle initial mass
- RCS modifications roughly double final efficiency and increase variation by almost an order of magnitude

	LEU LH2 NTP	HEU LH2 NTP	LEU CH4 NTP	CHM LOX-LH2
$\eta_{exg}\ (max)$	47.63%	47.68%	41.20%	31.83%
$\eta_{exg}\ (total)$	10.61%	10.62%	9.69%	8.18%

Conclusions

- **Exergy Efficiency provides a system integration relationship for a spacecraft**
  - Provides a direct comparison of different types of vehicle systems
    - –LEU LH2 NTP (most efficient)
    - –HEU LH2 NTP (very close to LEU)
    - –LEU CH4 NTP (good efficiency)
    - –Chemical (LO2/LH2) (notably lower efficiency)
  - Provides an understanding of the main drivers in system efficiency including effects from the environment (Thermal, Vacuum, Gravity)
- **Exergy Efficiency provides a key Measure of Performance (MoP) for the interplanetary transfer system**